بسم الله الرحمن الرحيم

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① Limits:.

النهايات

① التعويض المباشر

EX:. $\lim_{x\to 2} \frac{3x+1}{x+3} = \frac{6+1}{2+3} = \frac{7}{5}$ ✓

EX:. $\lim_{x\to 3} \frac{x^2-9}{x+2} = \frac{9-9}{3+2} = \frac{0}{5} = 0$ ✓

[2] اذا كان التعويض المباشر يعطي $\frac{\text{صفر}}{\text{صفر}}$ هنالك ما كتابه

(أ) حلل - تحضر - تفحص

EX:. $\lim_{x\to 3} \frac{x^2-9}{x-3}$ $\frac{9-9}{3-3} = \frac{0}{0}$

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 3+3 = 6$

EX:. $\lim_{x\to 2} \frac{x^2-4}{x-2} = $ $\frac{0}{0}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 2+2 = 4$

EX:. $\lim_{x\to 3} \frac{x^2-x-6}{x^2-8x+15}$ $\frac{0}{0}$

$= \lim_{x\to 3} \frac{(x+2)\cancel{(x-3)}}{\cancel{(x-3)}(x-5)} = \frac{3+2}{3-5} = \frac{5}{-2}$

(c)

EX:. $\lim\limits_{x \to 2} \dfrac{x^2 + 2x - 8}{x^2 - 4}$

$= \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x+4)}{\cancel{(x-2)}(x+2)} = \dfrac{2+4}{2+2} = \dfrac{6}{4} = \dfrac{3}{2}$

تحليل فرق المكعبين :

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

EX:. $x^3 - 8 = x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$

EX: $x^3 - 27 = x^3 - 3^3 = (x-3)(x^2 + 3x + 9)$

EX: $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^3 - 8} = \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2 + 2x + 4)}$

$= \dfrac{2+2}{4+4+4} = \dfrac{4}{12} = \dfrac{1}{3}$

EX:. $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x^3 - 27} = \lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x^2 + 3x + 9)}$

$\dfrac{3+3}{9+9+9} = \dfrac{6}{27} = \dfrac{2}{9}$

EX:. $\lim\limits_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3} = \lim\limits_{x \to 9} \dfrac{\cancel{(\sqrt{x} - 3)}(\sqrt{x} + 3)}{\cancel{(\sqrt{x} - 3)}}$

$= \sqrt{9} + 3 = 3 + 3 = 6$

EX:. $\lim\limits_{x \to 25} \dfrac{\sqrt{x} - 5}{x - 25} = \lim\limits_{x \to 25} \dfrac{\cancel{\sqrt{x} - 5}}{\cancel{(\sqrt{x} - 5)}(\sqrt{x} + 5)}$

$= \dfrac{1}{\sqrt{25} + 5} = \dfrac{1}{5+5} = \dfrac{1}{10}$

ب) الضرب بالمرافق:

في حالة وجود جذر تربيعي يكون

$$\boxed{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = a-b}$$

EX:. $$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{(1+x)-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(\sqrt{1+x}+\sqrt{1-x})} = \frac{2}{\sqrt{1+0}+\sqrt{1-0}}$$

$$= \frac{2}{1+1} = \frac{2}{2} = \underline{\underline{1}}$$

EX:. $$\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3}$$

$$= \lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3} \cdot \frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}$$

$\sqrt{9}$

$$= \lim_{x\to 3} \frac{x+6-9}{(x-3)(\sqrt{x+6}+3)}$$

$$= \lim_{x \to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+6}+3)} = \frac{1}{\sqrt{3+6}+3} = \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3} = \frac{1}{6}$$

$\boxed{٣}$ اذا كان $x \to \infty$: نقسم البسط والمقام على x بأكبر أس

EX:.
$$\lim_{x \to \infty} \frac{3x^2+2x-1}{5x^2+4}$$

$$= \lim_{x \to \infty} \frac{\frac{3x^2}{x^2}+\frac{2x}{x^2}-\frac{1}{x^2}}{\frac{5x^2}{x^2}+\frac{4}{x^2}} = \lim_{x \to \infty} \frac{3+\frac{2}{x}-\frac{1}{x^2}}{5+\frac{4}{x^2}}$$

$$= \frac{3+\frac{2}{\infty}-\frac{1}{\infty^2}}{5+\frac{4}{\infty^2}} = \frac{3+0-0}{5+0} = \frac{3}{5}$$

EX:.
$$\lim_{x \to \infty} \frac{2x^2-1}{5x^3-2x+4}$$

$$= \lim_{x \to \infty} \frac{\frac{2x^2}{x^3}-\frac{1}{x^3}}{\frac{5x^3}{x^3}-\frac{2x}{x^3}+\frac{4}{x^3}} = \lim_{x \to \infty} \frac{\frac{2}{x}-\frac{1}{x^3}}{5-\frac{2}{x^2}+\frac{4}{x^3}}$$

$$= \frac{\frac{2}{\infty}-\frac{1}{\infty^3}}{5-\frac{2}{\infty^2}+\frac{4}{\infty^3}} = \frac{0-0}{5-0+0}$$

$$= \frac{0}{5} = 0$$

EX:. $$\lim_{x\to\infty} \frac{2x^3+2}{3x^2-5x+7}$$

$$= \lim_{x\to\infty} \frac{\frac{2x^3}{x^3}+\frac{2}{x^3}}{\frac{3x^2}{x^3}-\frac{5x}{x^3}+\frac{7}{x^3}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{2}{x^3}}{\frac{3}{x}-\frac{5}{x^2}+\frac{7}{x^3}} = \frac{2+\frac{2}{\infty^3}}{\frac{3}{\infty}-\frac{5}{\infty^2}+\frac{7}{\infty^3}}$$

$$= \frac{2+0}{0-0+0} = \frac{2}{0} = \infty$$

$\frac{0}{a}=0$	$\frac{a}{0}=\infty$	$\frac{a}{\infty}=0$

* نظرية :.

$$\lim_{x\to\infty}\left[1+F(x)\right]^{g(x)} = e^{\lim_{x\to\infty} F(x)\cdot g(x)}$$

EX:. $$\lim_{x\to\infty}\left[1+\frac{1}{x}\right]^{3x} = e^{\lim_{x\to\infty}\left(\frac{1}{x}\right)(3x)}$$

$$= e^{\lim_{x\to\infty} 3} = e^{3}$$

$$\boxed{\lim_{x\to a} C = C}$$

EX $\lim\limits_{x \to \infty} \left[1 + \dfrac{2}{x^2}\right]^{\frac{x^2}{3}}$

$$= e^{\lim\limits_{x \to \infty} \left(\frac{2}{x^2}\right)\left(\frac{x^2}{3}\right)} = e^{\frac{2}{3}}$$

EX:. $\lim\limits_{x \to \infty} \left[1 - \dfrac{2}{x}\right]^{\frac{x}{3}}$

$$= \lim_{x \to \infty} \left[1 + \frac{-2}{x}\right]^{\frac{x}{3}} = e^{\lim\limits_{x \to \infty} \left(\frac{-2}{x}\right)\left(\frac{x}{3}\right)}$$

$$= e^{\frac{-2}{3}}$$

EX:. $\lim\limits_{x \to \infty} \left[\dfrac{x + 4}{x + 2}\right]^{3x}$

$$= \lim_{x \to \infty} \left[\frac{\frac{x}{x} + \frac{4}{x}}{\frac{x}{x} + \frac{2}{x}}\right]^{3x} = \lim_{x \to \infty} \left[\frac{1 + \frac{4}{x}}{1 + \frac{2}{x}}\right]^{3x}$$

$$= \lim_{x \to \infty}$$

$$\frac{\lim\limits_{x \to \infty} \left(1 + \frac{4}{x}\right)^{3x}}{\lim\limits_{x \to \infty} \left(1 + \frac{2}{x}\right)^{3x}} = \frac{e^{\lim\limits_{x \to \infty} \left(\frac{4}{x}\right)(3x)}}{e^{\lim\limits_{x \to \infty} \left(\frac{2}{x}\right)(3x)}}$$

$$= \frac{e^{12}}{e^{6}} = e^{12-6} = e^{6}$$

تدريب :

Find $\lim\limits_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x-2}$

$$= \lim_{x \to 2} \frac{\frac{2-x}{2x}}{x-2} = \lim_{x \to 2} \frac{\frac{-\cancel{(x-2)}}{2x}}{\cancel{x-2}}$$

$$= \frac{-1}{2(2)} = \frac{-1}{4}$$

٤) نهاية الدالة المثلثية :

① $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$ ٤ $\lim\limits_{x \to 0} \dfrac{\tan x}{x} = 1$

$$\lim_{x \to 0} \frac{\sin ax}{ax} = 1$$

EX:.

$$\lim_{x \to 0} \frac{\sin 3x}{2x} = \lim_{x \to 0} \frac{3}{2} \frac{\sin 3x}{3x}$$

$$= \frac{3}{2} \lim_{x \to 0} \frac{\sin 3x}{3x}$$

$$= \frac{3}{2}(1) = \frac{3}{2}$$

EX:. $$\lim_{x\to 0} \frac{\tan 2x}{5x} = \lim_{x\to 0} \frac{2}{5}\,\frac{\tan 2x}{2x}$$

$$= \frac{2}{5} \lim_{x\to 0} \frac{\tan 2x}{2x} = \frac{2}{5}(1) = \frac{2}{5}$$

EX:. $$\lim_{x\to 0} \frac{\sin 3x}{\tan 5x} = \lim_{x\to 0} \frac{\frac{\sin 3x}{x}}{\frac{\tan 5x}{x}}$$

$$= \frac{\lim_{x\to 0} 3\,\frac{\sin 3x}{3x}}{\lim_{x\to 0} 5\,\frac{\tan 5x}{5x}} = \frac{3 \lim_{x\to 0} \frac{\sin 3x}{3x}}{5 \lim_{x\to 0} \frac{\tan 5x}{5x}}$$

$$= \frac{3(1)}{5(1)} = \frac{3}{5}$$

EX:. $$\lim_{x\to 0} \frac{x}{\sin x} = \lim_{x\to 0} \frac{\frac{x}{x}}{\frac{\sin x}{x}}$$

$$= \frac{\lim_{x\to 0} 1}{\lim_{x\to 0} \frac{\sin x}{x}} = \frac{1}{1} = 1$$

قاعدة مهمة :

① $1 - \cos 2x = 2\sin^2 x$

② $1 - \cos x = 2\sin^2 \frac{x}{2}$

مهم * EX: Find $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

$$= \lim_{x\to 0} \frac{2\sin^2 \frac{x}{2}}{x^2} = 2\lim_{x\to 0} \frac{\sin\frac{x}{2}}{x} \cdot \frac{\sin\frac{x}{2}}{x}$$

$$= 2 \lim_{x\to 0} \frac{\sin\frac{x}{2}}{2\,\frac{x}{2}} \cdot \lim_{x\to 0} \frac{\sin\frac{x}{2}}{2\,\frac{x}{2}}$$

$$= 2\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \tfrac{1}{2}$$

Sand wich نظرية السااندوتش

if $F(x) \le g(x) \le h(x)$

and if $\lim\limits_{x\to a} F(x) = L = \lim\limits_{x\to a} h(x)$

then $\lim\limits_{x\to a} g(x) = L$

* $\boxed{-1 \le \sin x \le 1}$

EX: Find $\lim\limits_{x\to 0} x^2 \sin\frac{1}{x}$

<u>Solution</u> $-1 \le \sin\frac{1}{x} \le 1$

بالضرب في x^2 :

$$-x^2 \le x^2 \sin\frac{1}{x} \le x^2$$

$$\lim_{x \to 0} -x^2 = -0^2 = 0$$

$$\lim_{x \to 0} x^2 = 0^2 = 0$$

$$\therefore \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$$

Ex: Find $\lim_{x \to 0} \frac{1}{x^2} \sin x^2$

Solution

$$-1 \leq \sin x^2 \leq 1$$

بالضرب في $\frac{1}{x^2}$

$$\Rightarrow -\frac{1}{x^2} \leq \frac{1}{x^2} \sin x^2 \leq \frac{1}{x^2}$$

$$\lim_{x \to 0} \frac{-1}{x^2} = \frac{-1}{0} = \infty$$

$$\lim_{x \to 0} \frac{1}{x^2} = \frac{1}{0} = \infty$$

$$\therefore \lim_{x \to 0} \frac{1}{x^2} \sin x^2 = \infty$$

Continuous of Function : الإتصال :

① النهاية اليمنى واليسرى :

IF $F(X) = \begin{cases} \dfrac{X^2-4}{X-2} & X > 2 \\ X+2 & X < 2 \end{cases}$

Find $\lim\limits_{X \to 2} F(X)$

Solution

$$\lim_{X \to 2^+} F(X) = \lim_{X \to 2} \frac{X^2-4}{X-2} = \lim_{X \to 2} \frac{\cancel{(X-2)}(X+2)}{\cancel{X-2}}$$

$$= 2 + 2 = \boxed{4}$$

$$\lim_{X \to 2^-} F(X) = \lim_{X \to 2} (X+2) = 2+2 = \boxed{4}$$

$$\because \lim_{X \to 2^+} F(X) = \lim_{X \to 2^-} F(X) = 4$$

$$\therefore \lim_{X \to 2} F(X) = 4$$

EX:

$$F(X) = \begin{cases} \dfrac{\sin X}{3X} & X > 0 \\ \dfrac{X+2}{3} & X < 0 \end{cases}$$

Find $\lim\limits_{X \to 0} F(X)$

Solution

$$\lim_{x\to 0^+} F(x) = \lim_{x\to 0} \frac{\sin x}{3x}$$

$$= \lim_{x\to 0} \frac{\sin x}{3\ x} = \frac{1}{3} \lim \frac{\sin x}{x} = \frac{1}{3}$$

$$\lim_{x\to 0^-} F(x) = \lim_{x\to 0} \frac{x+2}{3} = \frac{0+2}{3} = \frac{2}{3}$$

$$\therefore \lim_{x\to 0^+} F(x) \neq \lim_{x\to 0^-} F(x)$$

$\therefore \lim_{x\to 0} F(x)$ Does not Exist

(c) دالة القيمة المطلقة :

$$|x-a| = \begin{cases} x-a & x > a \\ -(x-a) & x < a \end{cases}$$

EX:. Find $\lim_{x\to 2} \frac{|x-2|}{x-2}$

Solution

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} & x > 2 \\ \frac{-(x-2)}{x-2} & x < 2 \end{cases}$$

$$= \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

$$\lim_{x \to 2^+} F(x) = \lim_{x \to 2} 1 = 1$$

$$\lim_{x \to 2^-} F(x) = \lim_{x \to 2} -1 = -1$$

غير متساويتين

$$\therefore \lim_{x \to 2^+} F(x) \neq \lim_{x \to 2^-} F(x)$$

$\therefore \lim_{x \to 2} F(x)$ does not Exist

٣ الاتصال :

if $F(x)$ is a Function then

$F(x)$ is Continuous at $x = a$ if

$$\lim_{x \to a^+} F(x) = \lim_{x \to a^-} F(x) = F(a)$$

٪١٠٠

EX:. Discus if $F(x)$ is continuous at $x = 0$ if

$$F(x) = \begin{cases} \dfrac{1 - \cos x}{x^2} & x \neq 0 \\[2ex] \dfrac{1}{2} & x = 0 \end{cases}$$

(3)

Solution

$$\lim_{x\to 0} F(x) = \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}}{x^2}$$

$$= 2\lim_{x\to 0} \frac{\sin\frac{x}{2}}{x} \cdot \frac{\sin\frac{x}{2}}{x}$$

$$= 2\lim_{x\to 0} \frac{\sin\frac{x}{2}}{x} \cdot \lim_{x\to 0} \frac{\sin\frac{x}{2}}{x}$$

$$= 2\lim_{x\to 0} \frac{\sin\frac{x}{2}}{2\frac{x}{2}} \cdot \lim_{x\to 0} \frac{\sin\frac{x}{2}}{2\frac{x}{2}} = 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$F(0) = \frac{1}{2}$$

$$\because \lim_{x\to 0} F(x) = F(0)$$

$\therefore$ $F(x)$ is continuous

EX:. Discus ....

$$F(x) = \begin{cases} \frac{x^2-9}{x-3} & x > 3 \\ 6 & x = 3 \\ x+3 & x < 3 \end{cases}$$

at $x = 3$

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Solution

$$\lim_{x\to 3^+} F(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$$

$$\lim_{x\to 3^-} F(x) = \lim_{x\to 3} (x+3) = 3+3 = 6$$

$$F(3) = 6$$

$$\because \lim_{x\to 3^+} F(x) = \lim_{x\to 3^-} F(x) = F(3)$$

$\therefore$ $F(x)$ is Continuous

EX:.

$$F(x) = \frac{|x-4|}{x-4} \qquad \text{at } x=4$$

Solution

$$F(x) = \frac{|x-4|}{x-4} = \begin{cases} \frac{x-4}{x-4} & x \geq 4 \\ \frac{-(x-4)}{x-4} & x < 4 \end{cases} = \begin{cases} 1 & x \geq 4 \\ -1 & x < 4 \end{cases}$$

$$\left.\begin{aligned} \lim_{x\to 4^+} F(x) &= \lim_{x\to 4} 1 = 1 \\ \lim_{x\to 4^-} F(x) &= \lim_{x\to 4} -1 = -1 \end{aligned}\right\} \neq$$

$\therefore$ $F(x)$ is not Continuous